Question Booklet No. : .....

ENTRANCE TEST-2025

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

MATHEMATICS

Total Questions : 60
Time Allowed : 70 Minutes

Question Booklet Series **A**

Entrance Test Roll No. : | | | | | | |

Important Instructions for Candidates :

1. Candidates shall compulsorily use only **blue/ black ball point pen.** In no case gel/ink pen or pencil should be used.
2. Compulsorily write your **entrance test roll number** in the space provided at the top of this page of the question booklet.
3. Fill up the necessary information in the spaces provided on OMR Answer Sheet including **Question Booklet Number** and **Question Booklet Series**.
4. OMR Answer Sheet has an original copy and a candidate's copy glued beneath it at the top. While making entries in the original copy, candidate should ensure that the **two copies are aligned properly** so that the entries made in the original copy against each item are exactly copied in the candidate's copy.
5. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
6. **Choose only one correct/most appropriate response** for each question among the options A, B, C and D and darken the circle of the appropriate response completely. Incompletely darkened circle is not correctly read by the OMR scanner and no complaint to this effect shall be entertained.
7. **Do not darken more than one circle of option for any question. A question with more than one darkened response shall be considered wrong.**
8. **There will be negative marking for wrong answers. Each wrong answer will lead to deduction of 0.25 marks per wrong answer from the score.**
9. Only those candidates who obtain positive score in Entrance Test shall be eligible for admission.
10. Do not make any stray mark on the OMR sheet as this may lead to errors while scanning.
11. OMR answer sheet must be handled carefully and it should not be folded or mutilated, as in such case it will not be properly evaluated by the scanning machine.
12. Use of Electronic gadgets like calculator, mobile, smart watch, blue tooth etc. is strictly prohibited.
13. Rough work, if any, should be done on the blank sheets provided with the question booklet.
14. Ensure that the OMR Sheet is signed by the Examinee as well as by the invigilator.
15. At the end of the examination, fold the OMR Sheet along the crease on the top and tear off the top strip to separate the Original OMR Sheet from the Duplicate Copy.
16. Compulsorily hand over the **Original OMR Answer Sheet** to the invigilator.
17. Candidate's can retain duplicate copy of the OMR, Question Booklet and Admit Card.
18. If any of the information in the Response Sheet/Question Paper has been found missing or not mentioned as stated above, the candidate is solely responsible for that lapse.
19. Any deficiency on the OMR shall be the responsibility of the candidate himself/herself.

1. $\lim_{x\to\infty} \frac{x}{\sqrt{4x^2+1}-1}$ is equal to:

(A) 1

(B) -1

(C) $\frac{1}{2}$

(D) ∞

2. The function $f(x) = \begin{cases} x; & 0 \le x \le \frac{1}{2} \\ 1-x; & \frac{1}{2} < x \le 1 \end{cases}$ at $x = \frac{1}{2}$ is:

(A) Continuous but not differentiable

(B) Differentiable

(C) Discontinuous

(D) None of these

3. If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, then $\frac{\partial u}{\partial x} = \beta \frac{\partial u}{\partial y}$,

where β is equal to:

(A) $\frac{y}{x}$

(B) $\frac{x}{y}$

(C) $-\frac{y}{x}$

(D) $-\frac{x}{y}$

4. Which one of the following is true for $x > 0$?

(A) $\frac{x}{1+x} < x < \log(1+x)$

(B) $\log(1+x) < \frac{x}{1+x} < x$

(C) $\frac{x}{1+x} < \log(1+x) < x$

(D) None of these

5. In the Maclaurin's series expansion of the function $f(x) = e^x \cos x$, the coefficient of x^3 is:

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

6. The function $\frac{\log x}{x} (x > 0)$ has:

(A) Minimum value = e

(B) Maximum value = e

(C) Minimum value $= \frac{1}{e}$

(D) Maximum value $= \frac{1}{e}$

7. The value of $\int_0^{\frac{\pi}{2}} \sin^4\theta \cos^6\theta \, d\theta$ is:

(A) $\frac{\pi}{256}$

(B) $\frac{3\pi}{256}$

(C) $\frac{3\pi}{512}$

(D) $\frac{5\pi}{1024}$

8. The value of the integral $\int \frac{\sqrt{x}}{x+1} dx$ is:

(A) $2\sqrt{x} - \tan^{-1} x + c$

(B) $2x - \tan^{-1}\sqrt{x} + c$

(C) $2\sqrt{x} + 2\tan^{-1}\sqrt{x} + c$

(D) $2\sqrt{x} - 2\tan^{-1}\sqrt{x} + c$

9. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$ is :

(A) $\frac{1}{2}$

(B) 2

(C) $\frac{3}{2}$

(D) 1

10. The integrating factor of $x\frac{dy}{dx} + (3x+1)y = xe^{-2x}$ is :

(A) xe^{3x}

(B) $3xe^x$

(C) xe^x

(D) x^3e^x

11. A particular solution of the differential equation $(D^3-3D^2+3D-1)\,y = e^x \cos 2x$ is :

(A) $-\frac{1}{8}e^x \sin 2x$

(B) $\frac{1}{8}e^x \sin 2x$

(C) $-\frac{1}{8}e^x \cos 2x$

(D) $\frac{1}{8}e^x \cos 2x$

12. The differential equation $2ydx - (3y-2x)dy = 0$ is :

(A) Exact and homogeneous but not linear

(B) Homogeneous and linear but not exact

(C) Exact and linear but not homogeneous

(D) Exact, homogeneous and linear

13. If the eigen values of a 3×3 matrix are 1, 2 and 3, then the characteristic polynomial of the matrix is :

(A) $\lambda^3 - 6\lambda^2 - 11\lambda - 6$

(B) $\lambda^3 + 6\lambda^2 + 11\lambda - 6$

(C) $\lambda^3 - 6\lambda^2 + 11\lambda - 6$

(D) $\lambda^3 - 6\lambda^2 + 11\lambda + 6$

14. If A and B are square matrices of the same order, then adj (AB) is equal to :

(A) adj(A)adj(A)

(B) adj(B) adj(A)

(C) adj(A) + adj(B)

(D) None of these

15. Which one of the following is not ture ?

(A) If A is a symmetric matrix, then adj(A) is also symmetric

(B) If A is a non-singular matrix of order n, then $|adj(A)| = |A|^{n-1}$

(C) Product of two idempotent matrices is an idempotent matrix

(D) Product of two non-singular matrices is a non-singular matrix

16. The system of equations

$2x + y = 5$

$x - 3y = -1$

$3x + 4y = k$

is consistent when k is :

(A) 1

(B) 3

(C) 6

(D) 10

17. The elements on the principal diagonal of a Skew-Hermitian matrix are :

(A) Purely real or purely imaginary

(B) Purely imaginary or zero

(C) Zero only

(D) Complex numbers

18. Which one of the following is not true ?

(A) The rank of a unit matrix of order n is n

(B) The rank of a non-singular matrix of order n is n

(C) If A is a matrix of order m × n, then Rank (A) $\geq \min\{m, n\}$

(D) The rank of a skew-symmetric matrix cannot be one

19. The distinct eigen values of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are :

(A) 0 and 1
(B) 0 and –1
(C) 1 and 2
(D) 0 and 2

20. For the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, A^{-1} is given by :

(A) $A^2 + 2A - 3I$
(B) $A^2 - 2A + 3I$
(C) $A^2 - 2A - I$
(D) $A^2 - 3A + I$

21. Which one of the following is not true ?

(A) A finite set has no limit point
(B) The set Z of integers have no limit point
(C) The set Q of rational numbers have no limit point
(D) The set $S = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$ has only one limit point 0 which does not belong to S

22. The value of the limit

$\lim_{n\to\infty} \frac{1}{n}\left[1 + 2^{1/2} + 3^{1/3} + \ldots + n^{1/n}\right]$ is :

(A) 1
(B) $\frac{1}{2}$
(C) 0
(D) ∞

23. The limit superior and limit inferior of the sequence

$\left\{(-1)^n\left(1 + \frac{1}{n}\right)\right\}$ are respectively :

(A) 1 and $\frac{1}{2}$
(B) 2 and –2
(C) $\frac{1}{3}$ and $-\frac{1}{3}$
(D) 1 and –1

24. The sequence $\langle S_n \rangle$,

where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$ is :

(A) Convergent
(B) Divergent
(C) Unbounded
(D) Bounded but not convergent

25. The series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \ldots, (x > 0)$, converges for :

(A) $x \geq \frac{1}{e}$
(B) $x < \frac{1}{e}$
(C) $\frac{2}{e} < x < \frac{3}{e}$
(D) $\frac{1}{e} \leq x \leq \frac{2}{e}$

26. The series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ is :

(A) Conditionally convergent
(B) Absolutely convergent
(C) Divergent
(D) None of these

27. The series $\sum \frac{n^2 - 1}{n^2 + 1} x^n, (x > 0)$:

(A) Converges for $x \leq 1$
(B) Diverges for $x \leq 1$
(C) Converges for $x \geq 1$
(D) Diverges for $x \geq 1$

28. Which one of the following is not true ?

(A) The sequence $\{n(-1)^n\}$ oscillates infinitely
(B) The sequence $\{1 + (-1)^n\}$ oscillates finitely
(C) The sequence $\left\{\frac{(-1)^n}{n}\right\}$ oscillates infinitely
(D) The sequence $\left\{\frac{(-1)^n}{n}\right\}$ has only one limit point

29. For what value of μ does the line $y = x + \mu$ touches the ellipse $9x^2 + 16y^2 = 144$?

(A) $\mu = \pm 3$

(B) $\mu = \pm 5$

(C) $\mu = \pm 7$

(D) $\mu = \pm 9$

30. The equation of the chord of contact of tangents of tangents drawn from (2, 3) to the parabola $y^2 = 4x$ is :

(A) $x - 3y - 4 = 0$

(B) $2x - y + 4 = 0$

(C) $2x - 3y + 4 = 0$

(D) $2x - 3y - 4 = 0$

31. The pole of the line $y = 2x$ with respect to the parabola $y^2 = 2x$ is :

(A) (0, 2)

(B) (0, 1)

(C) (1, 2)

(D) $\left(0, \frac{1}{2}\right)$

32. Sum of the squares of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

(A) $a^2 + b^2$

(B) $a + b$

(C) $a^2 - b^2$

(D) $\sqrt{a^2 + b^2}$

33. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then the equation of hyperbola is :

(A) $x^2 - y^2 = 16$

(B) $x^2 - y^2 = 8$

(C) $x^2 - y^2 = 32$

(D) $y^2 - x^2 = 16$

34. The condition for the diameters $y = m_1x$ and $y = m_2x$ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to be conjugate is :

(A) $m_1m_2 = \frac{a^2}{b^2}$

(B) $m_1m_2 = \frac{b^2}{a^2}$

(C) $\frac{m_1}{m_2} = \frac{a^2}{b^2}$

(D) $\frac{m_1}{m_2} = \frac{b^2}{a^2}$

35. Condition that the plane $\ell x + my + nz = 0$ may touch the cone $4x^2 - y^2 + 3z^2 = 0$ is :

(A) $3\ell^2 + 4m^2 - n^2 = 0$

(B) $3\ell^2 - m^2 + 4n^2 = 0$

(C) $3\ell^2 + 12m^2 + 4n^2 = 0$

(D) $3\ell^2 - 12m^2 + 4n^2 = 0$

36. If a right circular cone has three mutually perpendicular generators, then the semi-vertical angle is :

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\tan^{-1}\sqrt{2}$

(D) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

37. If a and b are integers such that g.c.d.(a,b)=1 then g.c.d. $(a + b, a - b)$ is :

(A) 1

(B) 2

(C) 1 or 2

(D) None of these

38. The remainder obtained by dividing $1! + 2! + 3! + ...+ 100!$ by 12 is :

(A) 3

(B) 5

(C) 7

(D) 9

39. Which one of the following is a reduced residue system (mod 9) ?
(A) {1, 2, 3, 4, 5, 6}
(B) {1, 2, 3, 4, 5, 7}
(C) {1, 2, 4, 5, 6, 7}
(D) {1, 2, 4, 5, 7, 8}

40. The number of solutions in integers of the equation $2x + 4y = 7$ is :
(A) Two
(B) Three
(C) Four
(D) None of these

41. The number of positive integers less than 720 and prime to it is :
(A) 42
(B) 120
(C) 192
(D) 208

42. Which one of the following is the order of 3 (modulo 11) ?
(A) 2
(B) 3
(C) 4
(D) 5

43. The number of primitive roots (modulo 25) is equal to :
(A) 5
(B) 6
(C) 7
(D) 8

44. Value of the Legendre symbol $\left(\frac{3}{19}\right)$ is :
(A) 1
(B) –1
(C) 2
(D) –2

45. The number of generators of a cyclic group of order 8 is :
(A) 1
(B) 2
(C) 3
(D) 4

46. In the group G = {2, 4, 6, 8} under multiplication (modulo 10), the identity element is :
(A) 6
(B) 8
(C) 4
(D) 2

47. The order of the permutation (1 2) (3 4 5) in S_5 is :
(A) 2
(B) 3
(C) 5
(D) 6

48. Up to isomorphism, the number of abelian groups of order 108 is :
(A) 12
(B) 8
(C) 6
(D) 4

49. Which one of the following statements is not true ?
(A) Every ideal of a ring is its subring
(B) Every subring of a ring is its ideal
(C) Every division ring is a simple ring
(D) Every field is a simple ring

50. Which one of the following is a ring without unity ?
(A) The set Z of integers under usual addition and multiplication
(B) The set E of even integers under usual addition and multiplication
(C) The set M of 2×2 matrices over integers under matrix addition and multiplication
(D) A division ring

51. In the ring E of even integers, the ideal (4) is :
(A) Maximal
(B) Prime
(C) Both maximal and prime
(D) Neither maximal nor prime

52. The number of homomorphisms from Q (ring of rational numbers) into itself is :
(A) 1
(B) 2
(C) 3
(D) Greater than 3

53. If $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}; & (x,y)\neq(0,0)\\ 0 \quad ; & (x,y)=(0,0)\end{cases}$ then :

(A) $f_x(0,0)=0=f_y(0,0)$

(B) $f_x(0,0)=1$, $f_y(0,0)=-1$

(C) $f_x(0,0)=1$, $f_y(0,0)=\dfrac{1}{2}$

(D) $f_x(0,0)=-1=f_y(0,0)$

54. The function $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2}; & x^2+y^2\neq 0\\ 0 \quad ; & x=y=0\end{cases}$ is :

(A) Continuous but not differentiable at (0,0)

(B) Continuous with continuous partial derivatives but is not differentiable at (0,0)

(C) Differentiable at (0,0)

(D) None of these

55. If $u=\dfrac{1}{\sqrt{x^2+y^2+z^2}}$, then $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}=$

(A) 0

(B) 1

(C) $\dfrac{1}{2}$

(D) 2

56. For the function $f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^2+y^2}; & (x,y)\neq(0,0)\\ 0 \quad ; & (x,y)=(0,0)\end{cases}$,

which of the following is true ?

(A) $f_{xy}(0,0)\neq f_{yx}(0,0)$ and the conditions of Schwarz theorem are satisfied

(B) $f_{xy}(0,0)=f_{yx}(0,0)$ and the conditions of Schwarz theorem are not satisfied

(C) $f_{xy}(0,0)=f_{yx}(0,0)$ and the conditions of Schwarz theorem are satisfied

(D) None of these

57. The Laplace transform of $f(t)=t^n$, n is a positive integer is :

(A) $\dfrac{1}{s^{n+1}}$

(B) $-\dfrac{1}{s^{n+1}}$

(C) $\dfrac{n!}{s^{n+1}}$

(D) $-\dfrac{n!}{s^{n+1}}$

58. The inverse Laplace transform of $\dfrac{s}{s^2+a^2}$ is :

(A) $\dfrac{e^{iat}+e^{-iat}}{2}$

(B) $\dfrac{e^{iat}-e^{-iat}}{2}$

(C) $\dfrac{e^{iat}+e^{-iat}}{2i}$

(D) $\dfrac{e^{iat}-e^{-iat}}{2i}$

59. The Fourier series for $f(x)=\sin^2 x$ defined over the range $-\pi\le x\le\pi$ is :

(A) $\dfrac{1}{2}+\dfrac{\cos 2x}{2}$

(B) $\dfrac{1}{2}-\dfrac{\cos 2x}{2}$

(C) $\cos 2x$

(D) $-\cos 2x$

60. The Fourier series of an odd periodic function contains :

(A) Only sine terms

(B) Only cosine terms

(C) Both sine and cosine terms

(D) Constant term only

ROUGH WORK

Sr. No. ..... 352 .....

ENTRANCE TEST-2024

SCHOOL OF PHYSICAL & MATHEMATICAL SCIENCES

MATHEMATICS

Question Booklet Series **D**

Total Questions : 60
Time Allowed : 70 Minutes

Roll No. : | | | | | | |

Instructions for Candidates :

1. Write your Entrance Test Roll Number in the space provided at the top of this page of Question Booklet and fill up the necessary information in the spaces provided on the OMR Answer Sheet.
2. OMR Answer Sheet has an Original Copy and a Candidate's Copy glued beneath it at the top. While making entries in the Original Copy, candidate should ensure that the two copies are aligned properly so that the entries made in the Original Copy against each item are exactly copied in the Candidate's Copy.
3. All entries in the OMR Answer Sheet, including answers to questions, are to be recorded in the Original Copy only.
4. Choose the correct / most appropriate response for each question among the options A, B, C and D and darken the circle of the appropriate response completely. The incomplete darkened circle is not correctly read by the OMR Scanner and no complaint to this effect shall be entertained.
5. Use only blue/black ball point pen to darken the circle of correct/most appropriate response. In no case gel/ink pen or pencil should be used.
6. Do not darken more than one circle of options for any question. A question with more than one darkened response shall be considered wrong.
7. There will be **'Negative Marking'** for wrong answers. Each wrong answer will lead to the deduction of 0.25 marks from the total score of the candidate.
8. Only those candidates who would obtain positive score in Entrance Test Examination shall be eligible for admission.
9. Do not make any stray mark on the OMR sheet.
10. Calculators and mobiles shall not be permitted inside the examination hall.
11. Rough work, if any, should be done on the blank sheets provided with the question booklet.
12. OMR Answer Sheet must be handled carefully and it should not be folded or mutilated in which case it will not be evaluated.
13. Ensure that your OMR Answer Sheet has been signed by the Invigilator and the candidate himself/herself.
14. At the end of the examination, hand over the OMR Answer Sheet to the invigilator who will first tear off the original OMR sheet in presence of the Candidate and hand over the Candidate's Copy to the candidate.

SEAL

1. The solution of the linear partial differential equation $2p + 3q = 1$ is given by :

(A) $f(2x - y, 3y - z) = 0$

(B) $f(3x - 2y, y - 3z) = 0$

(C) $f(2x, 3y, z) = 0$

(D) None of these

2. The 2nd order partial differential equation $x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 3$, is elliptic if :

(A) $y^2 > 12x$

(B) $y^2 < 12x$

(C) $y^2 < 4x$

(D) $y^2 > 4x$

3. Choose the correct statement with regard to two bounded sets A and B such that $A \subset B$

I. $\sup A \leq \sup B$

II. $\inf A \geq \inf B$

(A) Only I

(B) Only II

(C) Both I & II

(D) Neither I nor II

4. Any infinite subset of a countable set :

(A) is countable

(B) is uncountable

(C) either countable or uncountable

(D) does not exist

5. Choose the correct statement from the following :

(A) The supremum of a bounded set always belongs to the set

(B) The infimum of a bounded set always belongs to the set

(C) Both supremum and infimum of a bounded set always belong to the set

(D) None of these

6. For a function $f : A \to B$, if the range of f is uncountable, then domain of f is :

(A) finite

(B) countable

(C) uncountable

(D) either countable or uncountable

7. Which of the following is not a Cauchy sequence ?

(A) $\left\{\frac{(-1)^n}{n}\right\}$

(B) $\{(-1)^n\}$

(C) $\left\{\frac{1}{n!}\right\}$

(D) $\left\{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{(-1)^{n-1}}{n}\right\}$

8. For the given statements, choose the correct statement (s) from the following :

I. A monotonic increasing sequence which is not bounded above, diverges to infinity

II. A monotonic decreasing sequence which is not bounded below, diverges to minus infinity

(A) only I

(B) only II

(C) both I & II

(D) neither I nor II

9. The sequence $\{S_n\}$, with $S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$;

(A) converges to some finite number

(B) diverges finitely

(C) converges to 2

(D) diverges to infinity

10. For what value of a, the value of $\lim_{n \to \infty} 3\frac{n^{1/n}}{2} = a^{-1}$?

(A) 0

(B) 1

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

11. If the series $\sum(2a_n - 1)$ is convergent, then the $\lim_{n \to \infty} a_n = ?$

(A) 0

(B) 1/2

(C) 1

(D) none of these

12. If $\sum a_n$ is a convergent series of positive terms, then $\sum \frac{a_n}{1+a_n}$ is :

(A) oscillatory

(B) divergent

(C) convergent

(D) none of these

13. The $\lim_{n \to \infty} \frac{1^2 + 2^2 + ... + n^2}{n^3} = ?$

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) None of these

14. For the series with nth term $a_n = \frac{1}{n(n+1)}$, the sum $\Sigma a_n = ?$

(A) 0

(B) 1

(C) 1/2

(D) Infinity

15. Which of the following correctly defines Leibnitz rule of a function given by $f(\alpha) = \int_a^b f(x, \alpha)\, dx$, where a, b are constants ?

(A) $f'(\alpha) = \frac{\partial}{\partial \alpha} \int_a^b f(x, \alpha)\, dx$

(B) $f'(\alpha) = \frac{d}{d\alpha} \int_a^b f(\alpha)\, dx$

(C) $f'(\alpha) = \int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\, dx$

(D) $f'(\alpha) = \int_a^b \frac{d}{d\alpha} f(x, \alpha)\, dx$

16. Given $f(a) = \int_a^b \frac{\sin ax}{x}\, dx$, then what is the value of $f'(a)$?

(A) $\frac{\sin 3a}{a}$

(B) $\frac{3\sin a^3 - 2\sin a^2}{a}$

(C) $\frac{3\sin a^2 - 2\sin^2 a}{a}$

(D) $\frac{3\sin a^3 - 3\sin^2 a}{6a}$

17. The power series $\Sigma 3^{-n}(z-1)^{2n}$ converges if :

(A) $|z| \leq 3$

(B) $|z| < \sqrt{3}$

(C) $|z-1| < \sqrt{3}$

(D) $|z-1| \leq \sqrt{3}$

18. Which of the following is the radius of convergence of the series $\Sigma z^{n!}$?

(A) 0

(B) ∞

(C) 1

(D) a real number > 1

19. If the number of generators in a cyclic group of infinite order is k, then k = ?

(A) 1

(B) 2

(C) finite

(D) infinite

20. If α and β are the smallest positive integers which respectively denote the orders of non-cyclic and non-abelian groups, then

(A) $\alpha = 4$ and $\beta = 4$

(B) $\alpha = 4$ and $\beta = 6$

(C) $\alpha = 6$ and $\beta = 4$

(D) None of these

21. The number of even permutations in a symmetric group S_6 is :

(A) 3

(B) 6

(C) 360

(D) 720

22. Which of the following is not a group under multiplication ? (Given $\omega^3 = 1, i^2 = -1$) :

(A) $\left\{1, 5, \frac{1}{5}\right\}$

(B) $\{1, -1, i, -i\}$

(C) $\{1, \omega, \omega^2\}$

(D) Non-zero rational numbers

23. Which of the following is true for a group of prime order p ?

I. It has no non-trivial subgroups

II. It is always cyclic as well as abelian

(A) Only I

(B) Only II

(C) Both I & II

(D) Neither I nor II

24. In the group of integer modulo 6, $<Z_6, +>$, the inverse of an element $\overline{2} \in Z_6$ is :

(A) $\overline{1}$

(B) $\overline{2}$

(C) $\overline{3}$

(D) $\overline{4}$

25. The alternating group A_n is not simple for :

(A) $n = 3$

(B) $n = 4$

(C) $n = 5$

(D) $n \leq 5$

26. Which of the following is not a normal subgroup ?

(A) Kernel of a homorphism

(B) Alternating group in S_n

(C) Centralizer of a group

(D) Normalizer of an element in a group

27. Choose the correct statement(s) :

I. Every ideal is a subring

II. The ring of complex numbers is a field

(A) only I

(B) only II

(C) neither I nor II

(D) both I & II

28. The polynomial $p(x) = a_0 + a_1x + ... + a_n x^n$; over a UFD is said to be primitive if the gcd of $a_0, a_1, a_2, ..., a_n$ is (Given $a_0, a_2, ..., a_n$ are integers)

(A) 1

(B) 2

(C) 3

(D) 4

29. Given two ideals I and J of a ring R such that $I \subset J$, then union of I and J :

(A) is an ideal

(B) need not be an ideal

(C) is a maximal ideal of R

(D) is a prime ideal of R

30. For what value of k, $\frac{z}{<k>}$ is a field ?

(A) $k = 1$

(B) $k = 2$

(C) $k = 4$

(D) Any value of k

31. Given a differentiable function $f(x)$ in a closed interval $[2, 7]$ with $f(2) = 3$ and $f'(x) \leq 5$ for all values of x in $(2, 7)$, the maximum possible value of $f(x)$ at $x = 7$ is:

(A) 7

(B) 14

(C) 15

(D) 28

32. Let $f(x)$ be continuous and differentiable function for all real's such that $f(x + y) = f(x) - 3xy + f(y)$ and $\lim_{h \to 0} \frac{f(h)}{h} = 7$, then the value of $f'(x)$ is :

(A) $-3x + 7$

(B) $3x - 7$

(C) $2f(x) + 7$

(D) 7

33. If $u = \log xy$ where $x^2 + y^2 = 1$, then $\frac{du}{dx} = ?$

(A) $\frac{1}{x}$

(B) $\frac{1}{x} + \frac{1}{y}$

(C) $\frac{1}{x} - \frac{x}{y}$

(D) $\frac{1}{x} - \frac{x}{y^2}$

34. The nth derivative of the function $f(x) = x^n e^x$ at $x = 0$ is given by :

(A) $n!\, e^n$

(B) $n!$

(C) $(n + 1)!$

(D) None of these

35. In a Cycloid, $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, the angle ψ the tangent makes to the horizontal is :

(A) 2θ

(B) θ

(C) $\frac{\theta}{2}$

(D) $\frac{\theta}{3}$

36. For $r = ae^{\theta \cot \alpha}$, the tangent is inclined at—angle to the radius vector.

(A) 0

(B) π

(C) $\frac{\pi}{2}$

(D) a constant

37. The curvature of the function $f(x) = x^3 - x + 1$ at $x = 1$ is given by :

(A) $\frac{6}{5}$

(B) $\frac{3}{5}$

(C) $\frac{6}{5^{\frac{3}{2}}}$

(D) $\frac{3}{5^{\frac{3}{2}}}$

38. A double point on a curve is said to be node, if two tangents of the curves :

(A) Are real and coincident

(B) Have no real tangents

(C) Are real and distinct

(D) None of these

39. The value of k in Rolle's theorem for $f(x)=x^3-3x$ in the interval $[0, \sqrt{3}]$ holds :

(A) 1

(B) $\frac{1}{3}$

(C) -1

(D) $\frac{2}{3}$

40. If the Rolle's theorem holds for the function $f(x) = x^4 + ax^3 + bx$, in $-1 \le x \le 1$ and $f'\left(\frac{1}{2}\right)=0$, then the value of ab = ?

(A) -64

(B) -8

(C) -4

(D) -1

41. Using Cauchy's Mean value theorem for the functions $f(x)=e^x$, $g(x)=e^{-x}$ in the interval $[2, 3]$, the value of c is :

(A) 1.5

(B) 2.5

(C) 3

(D) 3.5

42. Given $f(x)=x^3-3x-1$ is continuous in the closed interval $\left[\frac{13}{7}, -\frac{11}{7}\right]$ and $f'(x)$ exists in the interval, $\left(\frac{13}{7}, -\frac{11}{7}\right)$ the value of k such that it lies in $\left(\frac{13}{7}, -\frac{11}{7}\right)$ is :

(A) 0

(B) -1

(C) 1

(D) ± 1

43. The value of the integral $\int \frac{dx}{x(x+1)}$ is :

(A) $\ln\left|\frac{x+1}{x}\right|+c$

(B) $\ln\left|\frac{x}{x+1}\right|+c$

(C) $\ln\left|\frac{x-1}{x}\right|+c$

(D) $\ln\left|\frac{x}{x+1}\right|+c$

44. The reduction formula for $\int \tan^n x\, dx$ is :

(A) $\frac{\tan^{n-1} x}{n-1} - \int \tan^{n-1} x\, dx$

(B) $\frac{\tan^{n-1} x}{n} - \int \tan^{n-2} x\, dx$

(C) $\frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$

(D) $\frac{\tan^{n-1} x}{n-1} + \int \tan^{n-2} x\, dx$

45. For what value of k, $\int \frac{dx}{x(x+2)} = k \ln \left|\frac{x}{x+2}\right| + c$?

(A) 1

(B) –1

(C) 2

(D) $\frac{1}{2}$

46. What is the value of $\int_0^{8\pi} |\sin x|\, dx$?

(A) 2

(B) 4

(C) 8

(D) 16

47. The particular integral of $(D^2 - 3D + 2)\, y = e^{5x}$ is :

(A) e^{5x}

(B) $\frac{1}{30} e^{5x}$

(C) $\frac{1}{12} e^{5x}$

(D) None of these

48. The complementary function of $y_2 - 3y_1 + 2y =$ $\cos 3x$ is :

(A) $c_1 e^x + c_2 e^{2x}$

(B) $c_1 e^{-x} + c_2 e^{-2x}$

(C) $c_1 e^{2x} + c_2 e^{-3x}$

(D) None of these

49. The value of $\frac{1}{D^2 + a^2} \sin ax = ?$

(A) $-\frac{x}{2} \cos ax$

(B) $-\frac{x}{2a} \cos ax$

(C) $-\frac{x}{2a} \sin ax$

(D) $\frac{x}{2a} \cos ax$

50. The solution y (x) of $(D^2 + 1)\, y = 0$ is :

(A) $A \cos x - B \sin x$

(B) $Ae^x + Be^{-x}$

(C) $A \cos x + B \sin x$

(D) $-A \cos x + B \sin x$

51. The solution of $p^2 - 9p + 18 = 0$ is given by :

(A) $(y - x - a)\,(y + x - a)$

(B) $(y - 3x - a)\,(y - 6x - a)$

(C) $(y + 3x - a)\,(y - x + a)$

(D) None of these

52. Which of the following is the solution of $\frac{dy}{dx} = y\ (\text{cosec } x - \cot x)$?

(A) $y = c \sec^2 \frac{x}{2}$

(B) $y = c \log \sin \frac{x}{2}$

(C) $y = c \log \frac{x}{2}$

(D) None of these

53. The singular solution of $y = px + p - p^2$ is :

(A) $y = \frac{x+1}{2}$

(B) $y = \frac{x+1}{4}$

(C) $y = \left(\frac{x+1}{2}\right)^2$

(D) Does not exist

54. By means of a substitution $u = y^3$, the equation $y = 3xp + 6y^2p^2$ reduces to :

(A) $u = x\frac{du}{dx} + \frac{2}{3}\left(\frac{du}{dx}\right)^2$

(B) $u = x\frac{du}{dx} + \frac{1}{3}\left(\frac{du}{dx}\right)^2$

(C) $u = x\frac{du}{dx} - \frac{2}{3}\left(\frac{du}{dx}\right)^2$

(D) None of these

55. For any positive integer n and a Bessel's function $J_n(x)$, choose the correct expression from the following:

(A) $J_{-n}(x) = (-1)^{n+1} J_n(x)$

(B) $J_{-n}(x) = (-1)^n J_n(x)$

(C) $J_{-n}(x) = -J_n(x)$

(D) $J_{-n}(x) = (-1)^n J_{n+1}(x)$

56. Which of the following determines a Bessel's differential equation ?

(A) $x^2 y_2 + xy + (x^2 - n^2) y = 0$

(B) $x y_2 + y + xy = 0$

(C) $x^2 y_2 + xy + (x^2 - 1) y = 0$

(D) All of them

57. If $P_n(x)$ is the Legendre polynomial, then $P_0(x) + P_1(x) = ?$

(A) 1

(B) x

(C) $1 + x$

(D) None of these

58. The Wronskian of $y_1 = x^2$ and $y_2 = x^2 \log x$ is :

(A) 0

(B) 1

(C) x

(D) x^3

59. Be eliminating arbitrary constants a, b and c from $z = a(x + y) + b(x - y) + abt + c$, the resulting partial differential equation is :

(A) $z = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial t} + \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial t}\right)$

(B) $z = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial t} - \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial t}\right)$

(C) $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = 4\left(\frac{\partial z}{\partial t}\right)$

(D) None of these

60. By eliminating arbitrary function from $z = f\left(\frac{y}{x}\right)$, the PDE is given by :

(A) $px + qy = 0$

(B) $z = px + qy$

(C) $px - qy = 0$

(D) None of these