Department of Mathematics, University of Kashmir

1-Year Masters Programme in Mathematics Entrance Syllabus for the year 2026
(NEP-2020)

UNIT-I

Limits, Indeterminate forms. Continuity and differentiability, Successive differentiation and Leibnitz theorem, Partial and total differentiation, Homogenous functions and Euler's theorem, Curvature and radius of curvature. Extrema of functions: maxima and minima, Bounded functions and their properties. Characteristics of continuous functions on closed intervals. The intermediate value theorem. Rolle's theorem. Lagrange's Mean value theorem, Cauchy's Mean value theorem. Taylor's theorem. Taylor's series. Maclaurin's series of some standard functions like $\sin x$, $\cos x$, e^x, $\log x$, $(1+x)^m$.

UNIT-II

Standard cases of partial fractions. Integration of rational and irrational functions. Reduction formulae of some standard functions like $\int \sin^n x\, dx$, $\int \cos^n x\, dx$, $\int \tan^n x\, dx$, $\int \sin^n x \cos^m x\, dx$, $\int \sin^m x \sin^n x\, dx$, $\int \cos^m x \cos^n x\, dx$.

Definite Integrals, definite integral as a limit of sum, differentiation under an integral sign, fundamental theorem of calculus. First order and higher degree differential equations, Solvable for x, y and p, symbolic operators and Clairaut's form of differential equations, linear homogeneous and non-homogeneous differential equations with constant coefficients. Method of variation of parameters, exact differential equation and Cauchy-Euler equations.

UNIT-III

Hermitian and skew-Hermitian matrices, Orthogonal and Unitary matrices, representation of a square matrix as $P + iQ$ where P is Hermitian and Q is skew-Hermitian, adjoint of a matrix, reversal law of transpose, inverse of a matrix, reversal law for the inverse of matrices under multiplication, homogeneous and non-homogeneous system of equations and their consistency. Inner product of vectors, length of a vector, normal vectors. Trace of a matrix, Characteristic and minimal equations of a matrix. Block matrices. Cayley Hamilton theorem. Eigenvalues and Eigenvector. Rank of a matrix and its relation with linear dependence and independence of rows and columns. Elementary transformations of a matrix.

UNIT-IV

Sequences of real numbers ($\mathbb{R}$): Bounded and unbounded sets, supremum and infimum (lub and glb), Order completeness of $\mathbb{R}$. Archimedean property. Rational and Irrational density theorems, Dedekind's property, Sequences of real numbers. Convergence of sequences. Limit points of a sequences. Bolzano-Weierstrass theorem for sequences, limit inferior and limit superior, Theorems on limits and convergence of sequences, Cauchy's criteria for convergence of sequences, Cauchy sequences, monotonic sequences, Nested Interval theorem.

Professor & Head
Department of Mathematics

UNIT-V

Infinite Series: Convergence and Divergence of a series, Necessary condition for convergence of a series, Cauchy's criteria for Convergence of a series. Geometric series. Series of positive terms. Tests for convergence: Comparison test, Cauchy's root test, D-Alembert's ratio test, Raabe's test. Logarithmic test and Integral test.

UNIT-VI

Parabola: Tangents and normals. pole and polar. pair of tangents from a point. equation of a chord of a parabola in terms of its middle point. parametric equations of a parabola.
Ellipse: Tangents and normals. pole and polar. parametric equations of ellipse. diameters, conjugate diameters and their properties. General second-degree equation in x and y conditions under which a general second degree equation represents a conic. determination of equation of the corresponding conic.

UNIT-VII

Hyperbola: Tangents and normal, equation of hyperbola referred to asymptotes as axes. rectangular and conjugate diameters and their properties.
Cone: Vertex. guiding curve. generator. equation of cone with vertex as origin or a given vertex and guiding curve. Condition that the general equation of the second degree should represent a cone, Necessary and sufficient conditions for a cone to have three mutually perpendicular generators.

UNIT-VIII

Number theory: Divisibility of integers. prime numbers, fundamental theorem of arithmetic. Euclid's division algorithm. GCD and LCM of integers and their properties. Euclid's first theorem. Linear Diophantine equations, Necessary and sufficient condition for solvability of linear Diophantine equations. Linear Congruences and their solutions. Euclid's Second theorem. Fermat numbers. Complete Residue System (CRS). Reduced Residue System (RRS). Fermat and Euler's theorems with applications. Number theoretic functions: Euler's ϕ-function. $\phi(mn) = \phi(m)\phi(n)$ where $(m, n) = 1$; $\sum_{d|n} \phi(d) = n$; $\phi(m) = m\prod(1 - \frac{1}{p})$ for $m > 1$. Chinese Remainder theorem and its applications.

UNIT-IX

Group theory: Equivalence relations, binary composition. Introduction to groups: Finite and infinite groups, Semi-groups. various properties of groups. subgroups and cosets. Cyclic groups, Generators and relations. Structure theorem, Lagrange's theorem. Examples of General linear groups, Symmetric and alternating groups, Normal subgroups. product of subgroups. counting principle, Quotient groups. homomorphism, Kernel of a homomorphism, Fundamental theorem of homomorphism. Isomorphism theorems, Conjugate elements, Normalizer of an element. Center of a group.

UNIT-X

Rings: Definition and examples. Elementary properties of rings. Rings with and without zero divisors. Integral domains. skew field and fields. sub rings and sub fields, Ideals: Prime and Maximal ideals. Quotient rings. Idempotent and Boolean rings. Homomorphism. Fundamental theorem and ring isomorphism.

UNIT-XI

Functions of several variables: Limit, continuity, and differentiability of a function of two variables, Mean value theorem, Sufficient conditions for continuity, Sufficient condition for differentiability, Explicit and implicit functions, functions, partial derivatives of higher order and Change in the order. Young's theorem and Schwarz's theorem.

UNIT-XII

Fourier Series: Fourier Series of periodic functions, Even and Odd functions, elementary functions, Properties. Euler's Formulae for Fourier Series. Laplace transform: Laplace transform as periodic functions. Dirac-Delta function, Inverse Laplace transform, Laplace transform for derivatives and integrals.

UNIT-XIII

Advanced Group and Ring Theory: Automorphisms. Cauchy's theorem for abelian groups. Permutation groups. Symmetric groups, Alternating groups. Simple groups, Sylow's theorems. Class equations. Direct products. Normal and composition series, Jordan-Holder theorem. Solvable groups.
Euclidean rings. Principal Ideal Rings (PIR). Unique Factorization Domains (UFD). Polynomial rings (Division algorithm. Irreducibility criteria. Eisenstein's criterion. Cyclotomic polynomials).

UNIT-XIV

Riemann-Stieltje's integrals: Upper and lower sums under refinement, Criterion for RS-integrability of continuous and monotonic functions, reduction of an RS-integral to a Riemann integral. Fundamental theorem of calculus for Riemann integrals. Improper integrals: Integration of unbounded functions with finite limit of integration, Tests for convergence. Inequalities: AM-GM. Cauchy Schwartz, Jensen. Holder and Minkowski.

UNIT-XV

Infinite series: Carleman's theorem. conditional and absolute convergence. Merten's theorem. Dirichlet's theorem, Riemann's rearrangement theorem. Young's form of Taylor's theorem. generalized second derivative. Sequences and series of functions: point wise and uniform convergence, Various tests for uniform convergence, R- integration and differentiation, Weierstrass approximation theorem.

UNIT-XVI

Metric spaces: Open and closed sets. Completeness in metric spaces. Baire's category theorem. Cantor's intersection theorem. Uniform continuity. Banach's contraction principle.
Topological spaces: Elementary properties. Kuratowski's axioms. Continuous mappings. Pasting lemma. Bases and sub bases, Countability and Separability. Product and weak topology, Compactness. sequential compactness and total boundedness in metric spaces. Tychnoff's theorem. Lebesgue's covering lemma. Separation axioms, Connectedness. Urysohn's lemma and Urysohn's metrization theorem.

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